Models of the Structure and Evolution of Protoplanetary Disks

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In the context of. . .

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Overview

Vocabulary

Translation for better understanding

protoplanetary disk	...	Protoplanetare Scheibe
pressure scale height	...	Druckskalenhöhe
mass surface density	...	hier: Säulendichte
speed of sound	...	Schallgeschwindigkeit
accretion rate	...	Zuwachsrate
collisional cross section	...	Wirkungsquerschnitt
drag force	...	Zugkraft
eddy	...	Wirbel

List of physical quantities

Physical quantities used for disk modeling

pressure scale height	...	$H_p = \frac{c_s}{\Omega_K}$	$[H_p] = [L]$
mass surface density	...	Σ	$[\Sigma] = [M \cdot L^{-2}]$
speed of sound	...	$c_s^2 = \frac{\partial P}{\partial \rho}$	$[c_s] = [L \cdot T^{-1}]$
kinematic viscosity	...	ν	$[\nu] = [L^2 \cdot T^{-1}]$
Keplerian angular velocity	...	$\Omega_K = \sqrt{\frac{GM_*}{R^3}}$	$[\Omega_K] = [T^{-1}]$

Overview

Mass evolution of star and disk

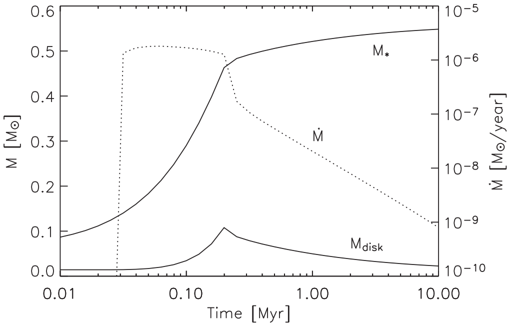

Abbildung: Mass evolution of star and disk, disk is formed at $t \approx 0.03$

Cross section of a planetary disk

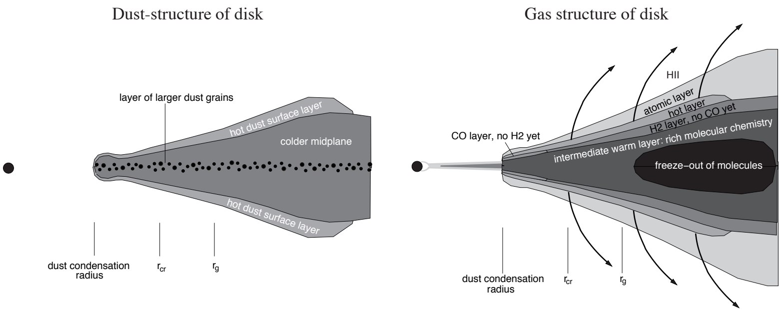

Abbildung: Structure of a flaring protoplanetary disk

Overview

Modeling of the disk

Captions

- **Formation and viscous evolution**
- Vertical structure
- Gas temperature and line spectra
- Photoevaporation by central star

Formation of dust in disk

We will focus on the vertical flow of dust.

Three major developments:

1. Settling of dust in protoplanetary disk
2. Vertical stirring of dust in protoplanetary disk
3. Mass accretation of protoplanetary disk

Settling of dust grains

We want to calculate the time scale t_{sett} in which dust grains settle:

Assumptions

- Grains experience drag forces with gas $\Rightarrow$ grains settle
- Time scale depends on surface-to-mass ration of the grain $\Rightarrow$ large grains settle faster
- Time scale also depends on surface density of gas $\Sigma(R)$
- Below z_{sett} equalized abundance

Settling of dust grains - friction time t_{fric}

Time scale t_{fric} in which a dust grain responds to the motion of gas

- m_{dust} . . . mass of a dust particle
- v_{sett} . . . relative velocity between gas and dust
- F_{fric} . . . friction force, depending on serveral properties of gas and dust

$$t_{\text{fric}} = \frac{m_{\text{dust}} v_{\text{sett}}}{|F_{\text{fric}}|}$$

Friction force

Common approach - the Epstein regime

$$F_{\text{fric}} = -\frac{4}{3}\rho_{\text{gas}}\sigma_{\text{dust}}v_{\text{sett}}c_s \qquad \text{with}$$

$$c_s = \sqrt{\frac{kT_{\text{gas}}}{\mu_{\text{gas}}m_{\text{p}}}} \qquad \text{isothermal speed of sound}$$

$$\sigma_{\text{dust}} = a^2\pi \qquad \text{collisional cross section}$$

with a being the radius of the dust grain ($< 1\,\text{cm}$)

Dust density

For $z \ll r$ (thin-disk-approximation)

$$\rho(z) = \frac{\Sigma(R)}{\sqrt{2\pi} H_p} e^{-\frac{z^2}{2H_p^2}}$$ with

$$H_p = \frac{c_s}{\Omega_K}$$ pressure scale height

$$\Omega_K = \sqrt{\frac{GM_*}{r^3}}$$ angular velocity

Settling speed v_{sett}

Equilize vertical forces $\Rightarrow v_{\text{sett}}$

$$
\begin{aligned}
F_{\text{grav}} &= -m_{\text{dust}}\Omega_K^2 z \\
F_{\text{fric}} &= -\frac{4}{3}\rho_{\text{gas}}\sigma_{\text{dust}}v_{\text{sett}}c_s \\
F_{\text{grav}} &= F_{\text{fric}} \\
\Rightarrow v_{\text{sett}} &= -\frac{3\Omega_K^2 z}{4\rho c_s}\frac{m_{\text{dust}}}{\sigma}
\end{aligned}
$$

Settling Time

Settling time dominated by local settling speed at height z

$$t_{\text{sett}} = \frac{z}{v_{\text{sett}}} = \frac{4}{3}\frac{\sigma}{m_{\text{dust}}}\frac{\rho c_s}{\Omega_K^2}$$

$$t_{\text{sett}}(0) = \frac{4}{3\sqrt{2\pi}}\frac{\sigma}{m_{\text{dust}}}\frac{\Sigma(R)}{\Omega_K}$$

Averaged vertical viscosity

There is a random vertical movement of dust caused by turbulences. We introduce a diffusion coefficient with the dimensions of kinematic viscosity $[L^2 \cdot T^{-1}]$.

Modeling approach with diffusion coefficient D_0

$$D_0 = \alpha c_s H_p$$

$$\alpha \approx 0.01 \qquad \text{for sufficiently ionized disk}$$

This model is called α-Disk model for protoplanetary disks.

Time scale t_{stir}

Modeling approach for the time scale t_{stir} before dust is effected by stirring

Stirring time t_{stir}

$$t_{\text{stir}} = \frac{z^2}{\frac{D_0}{\text{Sc}}}$$

$$\text{Sc}(z) = (1 + \text{St}) \qquad \text{Schmidt number}$$

$$\text{St}(z) = \frac{t_{\text{fric}}}{t_{\text{eddy}}} \qquad \text{Stokes number}$$

Stirring time scale t_{stir}

Stirring time scale t_{stir}

$$t_{\mathrm{stir}} = \frac{Sc}{\alpha\Omega_K}\frac{z^2}{H_p^2}$$

Linking stirring time scale t_{stir} and settling time scale t_{sett}

$$t_{\mathrm{stir}} \propto t_{\mathrm{sett}}$$

Settling-Stirring equalizing - dust depletion height z_{depl}

Weighing stirring time scale t_{stir} and settling time scale t_{sett}

$$t_{\text{stir}} = \xi t_{\text{sett}}$$

choice of ξ

- $\xi = 1$ gives the height $\frac{z_{\text{sett}}}{R}$ down to which typical grains tend to settle.
- $\xi = 100$ gives the depletion height $\frac{z_{\text{depl}}}{R}$ above which turbulences are ineffective.

Dust depletion height z_{depl}

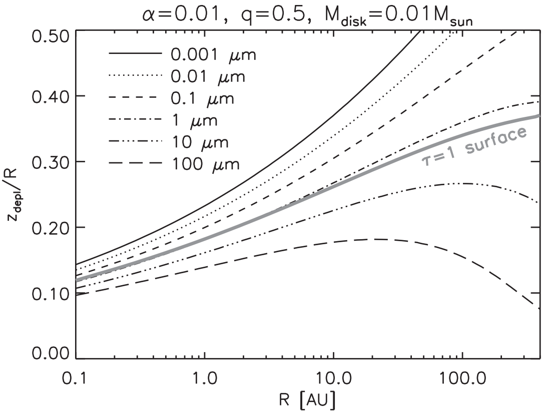

Abbildung: The dust depletion height z_{depl}

Overview

Artistic video of a protoplanetary disk

Our alternative to scientific calculations

http://www.youtube.com/watch?v=eYrQgLRc8Ms

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