The N-Body Problem

Some Inquest

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In the context of...

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Preface

Theory

Modeling a system with n degrees of freedom

An approach for a system with n degrees of freedom

$$\ddot{x} = F(x, \dot{x})$$

- $\vec{x}$ is a n-dimensional vector
- $F(x, \dot{x})$ is a Lipschitz-continuous function
- marginal conditions $x(t_0) = x_0$ und $\dot{x}(t_0) = v(t_0) = v_0$

Equation of motion

Differential equation for motion in Euclidean space

$$M\ddot{x} + C\dot{x} + F(x) = 0$$

- $M \in \mathbb{R}^{n \times n}$ is a symmetric, pos.def. mass matrix
- $C \in \mathbb{R}^{n \times n}$ is the matrix of friction
- $F(x) \in \mathbb{R}^{n}$ is a force

The N-body problem in $\mathbb{R}^3$

In $\mathbb{R}^3$ there are $n = 3N$ degrees of freedom

$$\begin{aligned} M\ddot{x} + C\dot{x} + F(x) &= 0 \\ M\ddot{x} + \nabla V(x) &= 0 \end{aligned}$$

- $x = (x_1^{(x)}, x_1^{(y)}, x_1^{(z)}, \dots x_N^{(x)}, x_N^{(y)}, x_N^{(z)})^T$
- $M = D(m_1, m_1, m_1, \dots m_n, m_n, m_n)$
- $C = 0$
- $V(x) = -\kappa \sum_{1 \leq i < j \leq N} \frac{m_i m_j}{|x_i - x_j|}$

The Hamilton function

Definition of Hamilton energy

$$H = \frac{1}{2} v^T M v + V(q) = \frac{1}{2} p^T M^{-1} p + V(q)$$

Hamilton for the N-body problem

$$\begin{aligned} H &= \sum_{1 \le i \le N} \frac{1}{2} m_i v_i^T v_i - \kappa \sum_{1 \le i < j \le N} \frac{m_i m_j}{|x_i - x_j|} \\ \dot{H} &= -v^T C v = 0 \end{aligned}$$

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Observables

It is essential

$$\dot{q} = \frac{\partial H(q,p)}{\partial p}$$
$$\dot{p} = -\frac{\partial H(q,p)}{\partial q}$$

Introduction of Lie-Algebra

$$\dot{f}(q,p) = \{f, H\} = H \neg f$$

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Angular momentum

Total angular momentum of system

$$L(q,p) = \sum_{1 \leq i \leq N} q_i(t) \times p_i(t)$$

Total angular momentum is invariant

$$H \neg L = 0_3$$

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Conservation of invariants

Projection to manifold

- In the configuration space the presumption of invariant observables $E(\hat{\xi}) = E(\xi_0) = E_0$ defines a manifold.
- Due to data error the numerical trajectory will not stay on this manifold.
- Projection $\xi \longrightarrow \hat{\xi}$ will correct this.

The projection method

Requirements for projection $\xi \to \hat{\xi}$

1. $E(\hat{\xi}) = E_0$
2. $||\xi, \hat{\xi}|| \to \min$

Projection for the N-body problem

variable position q, fixed momentum p

$$q \rightarrow q - \left(\frac{\partial \vec{E}}{\partial q}\right)^T . \left((\frac{\partial \vec{E}}{\partial q}).(\frac{\partial \vec{E}}{\partial q})^T \right)^{-1} . \left(\vec{E}(q,p) - \vec{E}_0 \right)$$

- $\vec{E}$ will be either Hamilton H or angular momentum $\vec{L}$
- combination of both causes troubles but can be serial
- there is a correlation between mapping H and $\vec{L}$.

Results

1. Preface
2. Theory
3. Results
4. Goodies

Goodies

A stable constellation without central mass